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Simple constructions for deformation in transpression/transtension zones

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Abstract—A simple geometrical construction is demonstrated for determining the orientations of the principal axes of the infinitesimal strain ellipsoid in tectonic zones where boundaries obliquely converge or diverge. The construction may also be used to determine the relative magnitudes of the principal axes and the ellipsoid shape, for infinitesimal and finite transpressional/transtensional strains. The analysis leads to a tightening of the definitions of the loosely used terms, compression, wrench, and extension. Applications of the simple construction and convenient graphical solution are illustrated by brief examples.

INTRODUCTION

THE RECOGNITION of the importance of oblique convergence and divergence of tectonic zones (transpression and transtension, Harland 1971) has been analysed by Sanderson & Marchini (1984). The objective of this short paper is to show how a simple geometrical construction, based on a modified Sanderson & Marchini model, permits the rapid determination of the orientations of the principal axes of the infinitesimal (or for practical purposes, small) strain ellipsoid. A simple graphical solution is also presented which enables the relative magnitudes of the axes and ellipsoid shape to be found for infinitesimal and finite strains where volume is conserved.

To simplify the discussion, and to emphasize its practical significance, the model is first presented in geometrical form, and some applications are discussed. A mathematical derivation of the construction is given in the Appendix.

GEOMETRICAL CONSTRUCTIONS

Method A: determination of the orientation of principal axes

If the direction of zone boundary displacement ($\underline{S}$) is known then the orientations of the principal axes of the infinitesimal sectional strain ellipse may be determined according to the construction illustrated in Fig. 1 as follows.

(i) Draw the zone boundaries (or boundary) in such an orientation that $\underline{S}$, the displacement vector points up the page.

(ii) Draw a circle, centred on the zone boundary. The radius is arbitrary, although half the normal separation places the constructed principal axes in the centre of the zone.

(iii) Construct a line normal to the zone boundary from the centre of the circle.

(iv) Identify the two points at the top and base of the circle which lie on a vertical diameter, i.e. parallel to $\underline{S}$.

(v) Project lines from the top and base of the circle through the intersection of the circle and the zone normal. The line from the top of the circle parallels the maximum principal axis of the infinitesimal sectional strain ellipse and that from the base parallels the minimum axis.

Method B: determination of the orientation of zone boundary displacement vectors

If the orientations of the maximum and/or minimum principal axes of the infinitesimal sectional strain ellipse can be inferred from first increment structures, e.g. tension gashes, stylolites, late folds and faults, etc. then the orientation of the zone displacement vectors may be determined as follows.

(i) Draw a circle, of arbitrary radius, centred on the zone boundary.

(ii) Project a line normal to the zone from the centre of the circle.

(iii) Draw lines parallel to the maximum and minimum principal axes of the infinitesimal sectional strain ellipse, so that their intersection coincides with that of the circle and the zone normal.

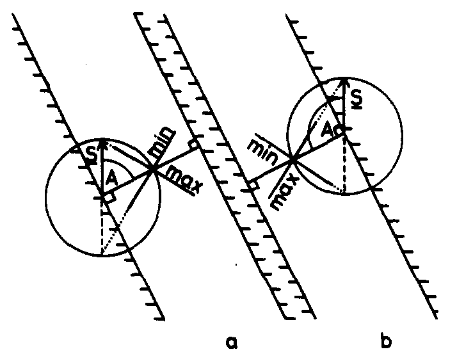

Fig. 1. A geometrical construction for (a) transpression and (b) transtension, which facilitates the identification of the orientations of the maximum and minimum principal axes of the infinitesimal sectional strain ellipse.

(iv) Project the maximum and minimum axes so that they intersect the circle, and draw a line between these intersections, through the circle's centre. This line parallels the displacement vector. The vector sense is from the intersection with the minimum axis towards that with the maximum axis.

GRAPHICAL SOLUTIONS FOR RELATIVE MAGNITUDES OF PRINCIPAL AXES

The relative magnitudes of the principal axes of the infinitesimal strain ellipsoid formed in the constant volume transpression/transtension model of Sanderson & Marchini (1984) can also be determined through the geometrical construction of Fig. 1. This is achieved by measuring the angle, (A) between $\underset{\sim}{S}$ and the zone normal. The infinitesimal relative magnitudes and ellipsoid shapes are a function of A and are outlined in Table 1 and Fig. 2(a). Knowing the relative magnitudes of the principal axes is important as they permit the tectonic regime in which they formed to be identified.

The nomenclature adopted in Table 1, where possible follows conventional schemes. The recognition of eight precise values of A, which bound distinctly different regimes (A = 0, 180°, and the sinistral and dextral systems where A = 70.5, 90, 109.5°) facilitates a tightening of the definition of vague terms such as compression, wrench, and extension. The adjective 'plane' is used to imply plane strain. The adjective 'general' is adopted to distinguish the three terms compression, wrench and extension from their vague common interpretation. General compression, wrench, and extension regimes are defined by the vertical orientations of the maximum, intermediate and minimum principal stretches respectively (cf. classification by Anderson (1951) with principal stresses). 'Axially symmetric' is preferred to 'uniaxial' as the deformation in these orientations is triaxial.

The critical angles and fields, detailed in Table 1, can be extended to finite strains, since the eight critical angles remain fairly stable with increasing strain (Fig. 2b). Four of the boundaries A = 0, 180° and the sinistral and dextral systems where A = 90°, remain fixed for all finite strains. The other four, where A = 70.5° (axially symmetric transpression or ASTP) and 109.5° (axially symmetric transtension or ASTT), migrate slowly towards 90° with increasing strain, (Fig. 2b). During constant volume zonal strains the fields of finite general compression and extension expand from their infinitesimal limits and that of general wrench contracts. This has two important consequences. First, for a constant direction of displacement A is fixed; thus only within the regime of infinitesimal general wrench (70.5° < A < 109.5°) can the relative magnitudes of finite and infinitesimal principal axes differ. This will only occur at fairly high strains except where A is very close to its critical values. Secondly, this stability of strain regime suggests that if axes swapping occurs in zones, it is more likely to be a result of changes in displacement direction at the zone boundary than due to progressive strain.

APPLICATIONS

Some applications of the construction in Fig. 1 and the graphical solution for determining deformation characteristics (Fig. 2) are illustrated by way of short examples.

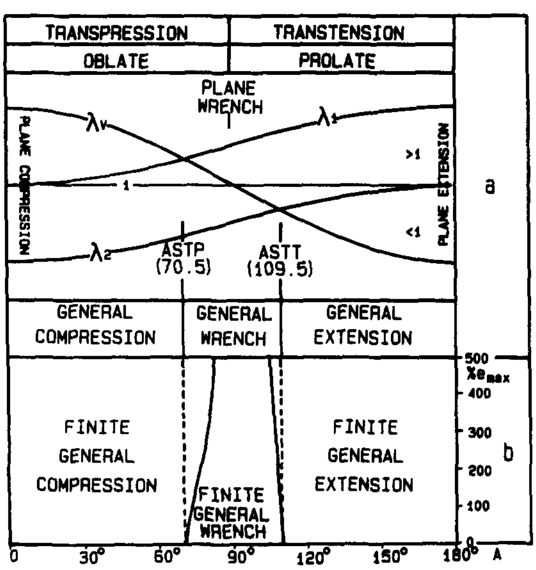

Fig. 2. A graphical solution for determining the characteristics of (a) the infinitesimal strain ellipsoid and (b & a) the finite ellipsoid. The orientations where axially symmetric transpression and transtension occur are shown by ASTP and ASTT, respectively. λ_v is the vertical principal quadratic elongation. $\lambda_1 > \lambda_2$ are the horizontal principal strains. e_{max} is the maximum finite extension.

Table 1. The relative magnitudes, implied tectonic regimes, and ellipsoid shapes for given values of A, as S → 0, in a vertical tectonic zone

A	Relative magnitudes	Tectonic regimes	Ellipsoid shape
Acute	see below	Transpression	Oblate
Obtuse	see below	Transtension	Prolate
0	$\lambda_v > \lambda_1 = 1 > \lambda_2$	Plane Compression	Planar
0 < A < 70.5°	$\lambda_v > \lambda_1 > 1 > \lambda_2$	General Compression	Oblate
70.5°*	$\lambda_v = \lambda_1 > 1 > \lambda_2$	ASTP	AS Oblate
70.5° < A < 90°	$\lambda_1 > \lambda_v > 1 > \lambda_2$	General Wrench (c-field)	Oblate
90°	$\lambda_1 > \lambda_v = 1 > \lambda_2$	Plane Wrench	Planar
90° < A < 109.5°	$\lambda_1 > 1 > \lambda_v > \lambda_2$	General Wrench (e-field)	Prolate
109.5° *	$\lambda_1 > 1 > \lambda_2 = \lambda_v$	ASTT	AS Prolate
109.5° < A < 180°	$\lambda_1 > 1 > \lambda_2 > \lambda_v$	General Extension	Prolate
180°	$\lambda_1 > \lambda_2 = 1 > \lambda_v$	Plane Extension	Planar

* Denotes the angles whose tangents are $\pm 2\sqrt{2}$. AS, TP, TT, c & e are abbreviations for; axially symmetric, transpression, transtension, compressional and extensional, respectively.

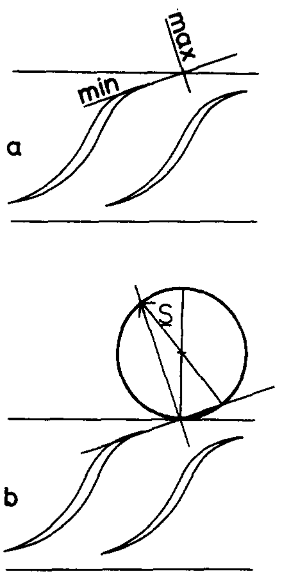

Fig. 3. A shear zone with sigmoidal tension gashes and identified orientations of the minimum and maximum principal axes of the infinitesimal sectional strain ellipse (a) and constructed zone displacement vector (b).

Example 1

Consider a shear zone with sigmoidal tension gashes (Fig. 3a). By assuming that the relatively undilated and unrotated ends of fractures parallel the orientation of the minimum principal axis of the infinitesimal sectional strain ellipse, Method B can be used to find the zone boundary displacement vector, (Fig. 3b). For further discussion of the evolution of extension fissures in shear zones see Ramsay & Huber (1983) where the special case of simple shear (A = 90°) is described in detail.

In this type of structure dilation may have occurred producing sectional area change probably without significant length change out of the page. Volume is not conserved in this shear zone; thus attempts to determine the relative magnitudes of the principal axes of the strain ellipsoid would be inappropriate.

Example 2

Consider a large tract of the upper crust between two major, en echelon rift zones; for example, between the Rhine and Bresse grabens (Fig. 4a). If it is assumed that the two major rifts open perpendicular to their boundary fault zones then by using Method A the orientations of the principal axes of the infinitesimal horizontal strain (or stress) ellipse may be predicted (Fig. 4(b)).

Furthermore if volume is taken to be conserved then by measuring the angle A, between S̲ and the zone normal (140° in this example), the relative magnitudes of the infinitesimal strain ellipsoid may be determined using Fig. 2(a). In this case the infinitesimal strain is transtensional and thus the ellipsoid is prolate in shape. The tract is undergoing general extension. The orientations of the finite strain axes are indeterminable by the constructions presented. However their relative magnitudes, and ellipsoid shape may be found from

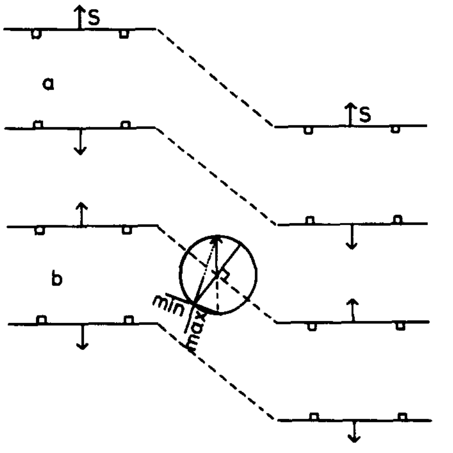

Fig. 4. A tract of crust between two major rift zones (a) and constructed orientations of the minimum and maximum principal axes of the infinitesimal sectional strain ellipse (b).

Fig. 2(a), because for all values of the maximum finite extension (e_{max}) in the zone with A = 140° the infinitesimal general extension produces finite general extension (Fig. 2b).

Example 3

Consider folds developing in a lateral tip zone (Coward & Potts 1983). If the displacement vector for the thrust sheet can be ascertained (Fig. 5a), then using Method A, the orientations of the principal axes of the infinitesimal horizontal strain ellipse in the lateral tip zone may be predicted (Fig. 5b). Fold axes might be expected to initiate parallel to the maximum principal axis. In this example A = 80°. Therefore, from Fig. 2(a) the infinitesimal strain is transpressional, and the ellipsoid is oblate in shape. The zone is undergoing general wrench (in the compressional field) and from Fig. 2(b) the finite strain ellipsoid will have the same characteristics if it can be shown that $e_{max} < 500\%$. If $e_{max} > 500\%$ then the deformation will result in finite general compression, i.e. the maximum finite stretch is vertical despite the maximum infinitesimal stretch being horizontal.

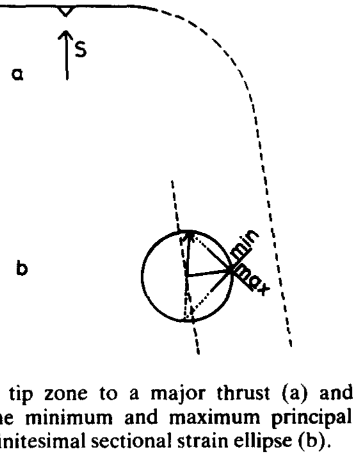

Fig. 5. A lateral tip zone to a major thrust (a) and constructed orientations of the minimum and maximum principal axes of the infinitesimal sectional strain ellipse (b).

CONCLUSIONS

A simple geometrical construction (Fig. 1) allows the rapid determination of the orientations of the principal axes of the infinitesimal strain (or stress) ellipsoid formed between obliquely converging or diverging tectonic boundaries. So simple is the construction that it can (at the expense of some accuracy) be drawn free hand in field notebooks. The graphical solution (Fig. 2) for determining infinitesimal and finite deformation characteristics is also more simple than existing complex multi- variable plots making its immediate use a practical reality for the geologist in the field or at his desk.

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REFERENCES

Anderson, E. M. 1951. *The Dynamics of Faulting*. Oliver & Boyd, Edinburgh.
Coward, M. P. & Potts, G. J. 1983. Complex strain patterns developed at the frontal and lateral tips to shear zones and thrust zones. *J. Struct. Geol.* **5**, 383–399.
Harland, W. B. 1971. Tectonic transpression in Caledonian Spitzbergen. *Geol. Mag.* **108**, 27–42.
Ramsay, J. G. & Huber, M. I. 1983. *The Techniques of Modern Structural Geology. Volume 1: Strain Analysis.* Academic Press, London.
Sanderson, D. J. & Marchini, W. R. D. 1984. Transpression. *J. Struct. Geol.* **6**, 449–458.

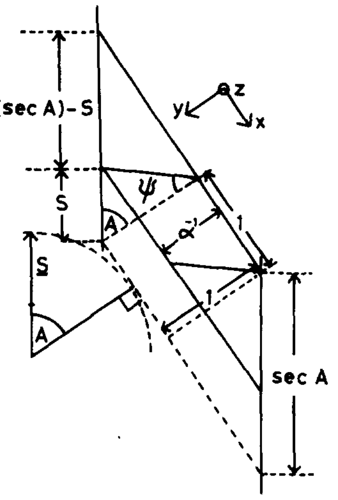

Fig. 6. A zone with boundaries tangential to a circle and parallel with $\underline{S}$ (to prevent lateral extrusion), showing the transformation of a unit square by stretch (α^{-1}) across the zone and shear (γ, where $\gamma = \tan \psi$) parallel to the zone. The circle, vector $\underline{S}$ and the zone boundary (parallel to x) are equivalent to those in Fig. 1.

APPENDIX

Assumptions

(i) In the triaxial strain analysis volume is conserved.

(ii) Lateral extrusion or intrusion of material from or into the tectonic zone is prohibited.

(iii) Material is allowed to thicken and thin vertically.

Consider the trend of a planar vertical zone boundary as a tangent to a horizontal circle, its position being fixed by the angle, A, between the zone's displacement vector, $\underline{S}$, and the normal to the zone. If the zone has unit width, and its tangential boundary is displaced, $\underline{S}$, then by considering the deformation of a unit square in the zone, the geometrical relationships of Fig. 6 can be derived. From Fig. 6, expressions for α^{-1} and γ can be derived in terms of $|\underline{S}|$ (or S) and A as follows, where stretch (α^{-1}) across the zone is simply the ratio of the deformed to original width of the zone,

$$\alpha^{-1} = 1 - \mathrm{S} \cos \mathrm{A}. \tag{1}$$

The shear strain parallel to the zone (γ) is given by

$$\gamma = \tan \psi = (\mathrm{S} \sin \mathrm{A})/(1 - \mathrm{S} \cos \mathrm{A}). \tag{2}$$

Following assumptions 1, 2 & 3, from Sanderson & Marchini (1984) the triaxial deformation D is given by

$$D = \begin{vmatrix} 1 & \alpha^{-1}\gamma & 0 \\ 0 & \alpha^{-1} & 0 \\ 0 & 0 & \alpha \end{vmatrix}. \tag{3}$$

From equation (3) the principal quadratic elongations and their orientations can be found in terms of α and γ and hence in terms of S and A. (See, for example, Ramsay & Huber 1983 Appendix B for discussion.) This yields

$$\lambda_1 \text{ or } \lambda_2 = p/2 \pm 1/2\,(p^2 - 4q^2)^{1/2} \tag{4a}$$

$$\text{where } p = 1 + \alpha^{-2}\gamma^2 + \alpha^{-2} \quad \text{and} \quad q = \alpha^{-1} \tag{4b \& c}$$

$$\lambda_v = \alpha^2 \tag{5}$$

$$\tan 2\theta' = 2\gamma/(\alpha^2 + \gamma^2 - 1). \tag{6}$$

The geometrical construction

The construction of Fig. 1 in the text assumes that the acute angle between the zone boundary (x-axis) and the maximum axis of the infinitesimal sectional ellipse is A/2. This is proved to be the case by rewriting (6) in terms of S and A (from 1 & 2), and letting $\mathrm{S} \to 0$.

The graphical solution

Numerical substitution in equations 4 & 5 for given values of S and A permits the identification of the deformation-type limits, for different finite strains and boundary orientations. In the limiting case as $\mathrm{S} \to 0$, the critical location angles (A′) for ASTP and ASTT are arctan ($\pm 2\sqrt{2}$). This can be demonstrated by defining γ in terms of α for the special case of axially symmetric strains

$$\gamma^2 = \alpha^4 - \alpha^2 + \alpha^{-2} - 1. \tag{7}$$

By substituting (7) in (6)

$$\lim_{S\to 0} \tan \mathrm{A}' = \lim_{\alpha\to 1} \tan \mathrm{A}' = \lim_{\alpha \to 1} \frac{2(\alpha^4 - \alpha^2 + \alpha^{-2} - 1)^{1/2}}{\alpha^4 + \alpha^{-2} - 2}$$

$$= \pm 2\sqrt{2}. \tag{8}$$